Theoretical Physics VI: Statistical Physics - Theory of Heat

Problem Set 11

due: 16. 1. 2008, 10:15 am

Problem 11.1 *Distribution functions of ideal quantum gases* (6 pts.)

(a) Show that the distribution function,

$$n(\varepsilon_i) = \frac{1}{e^{\beta(\varepsilon_i - \mu)} \mp 1}$$

with $-(+)$ applying to bosons (fermions) and μ denoting the chemical potential, is identical to the average occupation number of the single-particle state with energy ε_i.

(b) Sketch the Bose-Einstein, Fermi-Dirac and Boltzmann distribution functions and discuss their behavior.

Problem 11.2 *Fluctuations in ideal quantum gases* (5 pts.)
Calculate the root-mean-square deviation of the single-particle occupation numbers,

$$\left(\Delta n_i\right)^2 = \frac{\langle \hat{\mathbf{n}}_i^2 \rangle - \langle \hat{\mathbf{n}}_i \rangle^2}{\langle \hat{\mathbf{n}}_i \rangle^2},$$

for ideal Bose and Fermi gases.

Problem 11.3 *Trapped ideal Fermi gas* (6 pts.)
The single-particle energy levels for a set of non-interacting fermions in a harmonic oscillator potential are

$$\epsilon_n = \left(n + \frac{1}{2}\right)\hbar\omega \quad \text{with} \quad n = 0, 1, 2, \ldots$$

(a) Write the grand partition function Z_G and the grand potential Φ.

(b) Derive the expression for the number of particles $\langle N \rangle$ as a sum of Fermi-Dirac distribution functions.

(c) The sum for $\langle N \rangle$ can be changed to an integral when $\beta\hbar\omega \ll 1$. Derive an expression for the chemical potential, μ, as a function of β, $\hbar\omega$ and $\langle N \rangle$ in this high-temperature limit.

Hint:

$$\int \frac{dx}{e^x + 1} = -\ln\left(1 + e^{-x}\right)$$

Problem 11.4 *Ideal Fermi gas in a magnetic field* (8 pts.)
Compute the magnetization of an ideal gas of spin-$\frac{1}{2}$ fermions in the presence of a magnetic field. Assume that the fermions each have a magnetic moment, μ_e. Find an expression for the magnetization in the limit of weak magnetic field and $T \to 0$.