Theoretical Physics VI: Statistical Physics - Theory of Heat

Problem Set 8

due: 12. 12. 2007, 10:15 am

Problem 8.1 *Thermodynamics of the van der Waals gas* (5 pts.)

The equation of state for a van der Waals gas is

$$\left(p + \frac{aN^2}{V^2}\right)(V - Nb) = NRT\,,$$

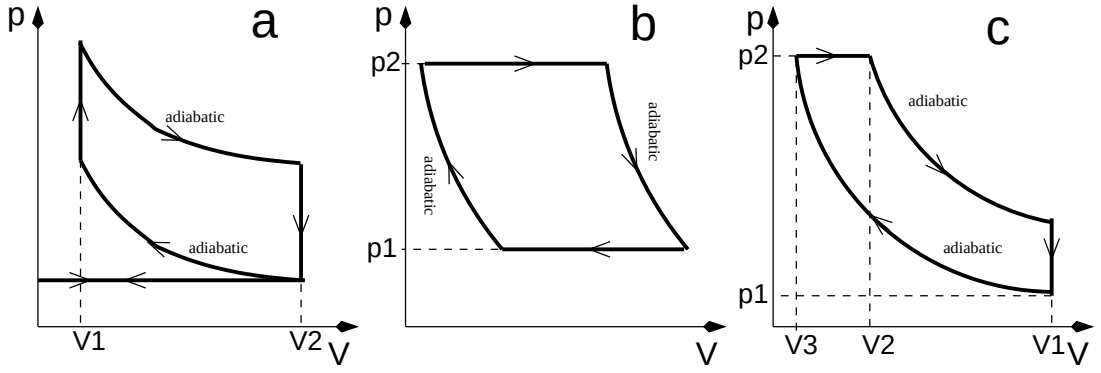
where a and b are constants which depend on the type of gas.

a) Assume that the heat capacity is $C_V = \frac{3}{2}NR$. Verify that C_V should indeed not depend on the volume. Show that the assumption $C_V = \frac{3}{2}NR$ allows to recover the expression of entropy and internal energy for the ideal gas in the limit $a = 0$, $b = 0$.

b) Compute the free energy.

Problem 8.2 *Cycles* (12 pts.)

Show that the efficiencies of the three cycles listed below for an ideal gas are as given in the list.



a) Otto cycle (figure a)

$$\eta = 1 - \left(V_2/V_1\right)^{\gamma-1}$$

b) Joule cycle (figure b)

$$\eta = 1 - \left(p_1/p_2\right)^{(\gamma-1)/\gamma}$$

c) Diesel cycle (figure c)

$$\eta = 1 - \frac{1}{\gamma}\frac{\left(V_2/V_1\right)^{\gamma} - \left(V_3/V_1\right)^{\gamma}}{\left(V_2/V_1\right) - \left(V_3/V_1\right)}$$

Problem 8.3 *Thermodynamic potentials* (8 pts.)

Compute the entropy, enthalpy, free energy, and Gibbs free energy of a paramagnetic substance in terms of their natural variables, given that the equation of state is

$$m = \frac{DH}{T}$$

and that the molar heat capacity at constant magnetization, $c_M = C_M/N$, is $c_M = c$. Here m denotes the molar magnetization and H the magnetic field, D and c are constants and T is the temperature.

Hint: Show first that the (molar) internal energy does not depend on magnetization.