Theoretical Physics VI: Statistical Physics - Theory of Heat

Problem Set 1

due: 24. 10. 2007, 10:15 am

Problem 1.1 *Stirling's formula* (4 pts.)
Prove that for large N

$$N! = \sqrt{2\pi N}\left(\frac{N}{e}\right)^N .$$

Hint: Use the identity $N! = \Gamma(N+1) = \int_0^\infty x^N e^{-x} dx$, and employ a Taylor expansion of $f(x) = N \ln x - x$ near the maximum.

Problem 1.2 *Binomial distribution* (8 pts.)
Show that

$$P_N(k) = \frac{N!}{(N-k)!k!} q^k (1-q)^{N-k} \tag{1}$$

becomes
(a) a Gaussian distribution for $N \gg 1$, and
(b) a Poissonian distribution for $q \ll 1$, $k \ll N$.
Plot an example for each case.

Problem 1.3 (3 pts.)
Fifteen kids go hiking. Five kids get lost, eight get sunburned, and six return home without problems. What is the probability that a sunburned kid got lost? What is the probability that a lost kid got sunburned?

Problem 1.4 (3 pts.)
Find the probability that in a class of N students all birthdays are different. How large should the class be to expect coinciding birthdays with a probability of at least 1/2?

Problem 1.5 (3 pts.)
Suppose that 5 out of 100 men and 25 out of 10000 women are color-blind. A color-blind person is chosen at random. Assuming the same number of men and women in the general population, what is the probability of this person being male?
optional: Can you make a hypothesis about how color-blindness is inherited?

Problem 1.6 (4 pts.)
Consider 1 mol of an ideal gas in a container. The identical particles are independently distributed and fill equal volumes of the container with equal probability. Calculate the probability that all particles are found in the left half of the container. Estimate the probability for a difference of more than 10^{-5} mol between the left and the right half of the container.