Solid State Theory
Exercise 12

Thermoelectric effects

Exercise 12.1 Domenicali equation

We write the phenomenological transport equations for the charge and heat currents in the form

$$\begin{aligned} \vec{J}_e &= \sigma\vec{\mathcal{E}} - \sigma Q \nabla T, \\ \vec{J}_q &= QT\vec{J}_e - \kappa \nabla T, \end{aligned} \tag{1}$$

where the electrochemical field is $\vec{\mathcal{E}} = -\nabla(\phi + \mu/e) = -\nabla\bar{\phi}$. The continuity equations for the charge and total energy current read

$$\begin{aligned} \dot{\rho} + \nabla \cdot \vec{J}_e &= 0, \\ \dot{u} + \nabla \cdot \vec{J}_u &= 0, \end{aligned}$$

where $\vec{J}_u = \vec{J}_q + \bar{\phi}\vec{J}_e$

a) Convince yourself that the form (1) is equivalent to the form given in the lecture.

b) Show that for $\dot{\rho} = 0$ the local heat production is $\dot{q} = \dot{u}$ with

$$\dot{u} = \frac{J_e^2}{\sigma} + \nabla \cdot (\kappa \nabla T) - T\nabla Q \cdot \vec{J}_e. \tag{2}$$

Interpret the different terms. Why is the thermolelectric heat reversible? In the stationary state Eq. (2) yields the so called *Domenicali* equation,

$$0 = \frac{J_e^2}{\sigma} + \nabla \cdot (\kappa \nabla T) - T\nabla Q \cdot \vec{J}_e, \tag{3}$$

which determines the temperature distribution in a sample for given boundary conditions.

Exercise 12.2 Thermoelectric cooler

Consider in the following a thermoelectric device of length l and cross section A connecting two reservoirs at temperatures T_0 and T_a (see Fig. 1).

a) Assume that in the operating temperature regime the transport coefficients σ, κ and Q are constant. For a given electrical current $I = J_e A$ compute the temperature distribution $T(x)$. Use Eq. (3) to show that

$$T(y) = T_0 + (T_a - T_0)\, y + \frac{RI^2}{2K} y(1-y), \quad y = \frac{x}{l}, \tag{4}$$

where R is the electrical resistance and K the thermal conductance.

b) Compute the heat current $AJ_q(x)$. What is the difference at the source $x = 0$ and at the sink $x = l$? What is the power supplied by the battery to maintain a stationary state?

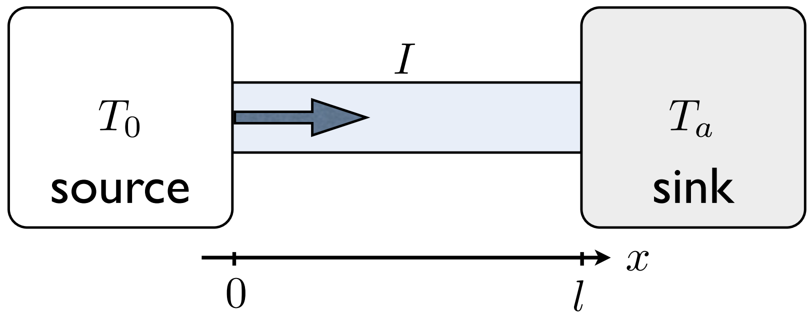

Figure 1: Schematic view of the setup for a thermoelectric cooler.

c) Assume that the reservoir at T_0 is thermally isolated. What is the maximal temperature difference which can be reached by optimizing the electrical current? Show that

$$(T_a - T_0)_{max} = \frac{1}{2} Z T_0^2, \quad Z = \frac{\sigma Q^2}{\kappa}. \tag{5}$$

Z is called the *figure of merit* of a given thermoelectric material. Calculate T_0 as function of T_a.

d) The coefficient of performance η of the cooling device is the ratio of the heat taken from the source per unit time and the power supplied by the battery. Optimize the performance with respect to the current I and show that

$$\eta_{max} = \frac{T_0}{T_a - T_0} \frac{\sqrt{1 + Z T_M} - \frac{T_a}{T_0}}{\sqrt{1 + Z T_M} + 1}, \quad T_M = \frac{T_0 + T_a}{2}. \tag{6}$$

What is the property of an efficient thermoelectric material? Compare η_{max} with the efficiency of a Carnot heat engine.