ETH
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Solid State Theory
Exercise 11

SS 08
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Exercise 11.1 Conductivity tensor

Calculate explicitly the conductivity tensor $\sigma_{\alpha\beta}$ for a dispersion relation of the form

$$\varepsilon_{\vec{k}} = \sum_{\alpha} \frac{\hbar^2 k_\alpha^2}{2m_\alpha} \tag{1}$$

in the static limit.

Exercise 11.2 Mean free path

a) Show for a system with cubic symmetry that the conductivity can be expressed as

$$\sigma = \frac{e^2}{h} \frac{S_F}{6\pi^2} \bar{l} \tag{2}$$

with S_F the area of the Fermi surface and $\bar{l}$ is the mean free path averaged over the Fermi surface,

$$\bar{l} = \frac{1}{S_F} \int dS |l_{\vec{k}}|. \tag{3}$$

b) In one-dimensional ballistic transport, every channel carries a conductance of e^2/h. Discuss eq.(2) under this point of view, especially the cases of empty or filled bands.

c) Table 1 lists the residual resistivity of Cu with different impurities. Try to explain the data qualitatively using the above derived formula.

Impurity	Resistivity (per 1% of impurity atoms) $\rho/(10^{-8}\Omega m)$
Be	0.64
Mg	0.6
B	1.4
Al	1.2
In	1.2
Si	3.2
Ge	3.7
Sn	2.8
As	6.5
Sb	5.4

Table 1: Residual resistivity of Cu for different impurities (From Landolt-Börnstein Tables, Vol 15, Springer, 1982)

Exercise 11.3 Magnetoresistance and Hall effect

We want to consider the electrical resistivity in the presence of a stationary magnetic field $\vec{B}$. The linearized Boltzmann equation in this case reads

$$-e\vec{v}_{\vec{k}} \cdot \vec{E}\frac{\partial f_0(\varepsilon_{\vec{k}})}{\partial \varepsilon_{\vec{k}}} = -\frac{g(\vec{k})}{\tau(\varepsilon_{\vec{k}})} + \frac{e}{\hbar}(\vec{v}_{\vec{k}} \times \vec{B})\frac{\partial g(\vec{k})}{\partial \vec{k}} \tag{4}$$

where $g(\vec{k}) = f(\vec{k}) - f_0(\vec{k})$.

a) Why is in (4) the magnetic field in first order not a driving field, contrary to the electric field?

b) Calculate for the case of $\vec{B} = (0, 0, B)$ the resistivity tensor $\hat{\rho} = \hat{\sigma}^{-1}$ and show that

   (i) the Hall resistance becomes independent of the scattering time τ and that

   (ii) the transverse magnetoresistance defined by

$$\Delta\rho_{xx}(B) = \rho_{xx}(B) - \rho_{xx}(0) \tag{5}$$

   is equal to zero.

   Hint: Consider independently the cases where the electric field points in the x, y and z direction, respectively, and use the ansatz

$$g(\vec{k}) = ak_x + bk_y. \tag{6}$$