## 8 Exercise - Introduction to Group Theory

## 8.1 (Symmetries of normal modes of vibrations)

### 8.1.1 HOCl

While we got 3 atoms, we expect $3 \cdot 3-6=3$ vibrational degrees of freedom. First we want to get the reducible representation for cartesian coordinate transformation. Therefore we first have to get the point group which in this case is $C_{s}$. The character table is given with

| $C_{s}$ | $E$ | $\sigma_{h}$ |
| :---: | :---: | :---: |
| $A^{\prime}$ | 1 | 1 |
| $A^{\prime \prime}$ | 1 | -1 |

From this we get the reducible representation

| $C_{s}$ | $E$ | $\sigma_{h}$ |
| :---: | :---: | :---: |
| $\Gamma$ | 9 | 1 |

Therefore we can reduce $\Gamma$ to

$$
\Gamma^{t o t}=5 a^{\prime}+4 a^{\prime \prime}
$$

Now subtracting 3 translational and 3 rotational degrees of freedom we get:

$$
\begin{aligned}
\Gamma^{v i b} & =\Gamma^{t o t}-\Gamma^{\text {trans }}-\Gamma^{\text {rot }} \\
& =5 a^{\prime}+4 a^{\prime \prime}-\left(2 a^{\prime}+a^{\prime \prime}\right)-\left(a^{\prime}+2 a^{\prime \prime}\right) \\
& =2 a^{\prime}+a^{\prime \prime}
\end{aligned}
$$

We got the same kind of modes like for $\mathrm{H}_{2} \mathrm{O}$ which was treated in the lecture, therefore we can identify the modes:

1. symmetric stretching mode (1) with $a^{\prime}, 2$. symmetric bending mode (2) with $a^{\prime}$ and 3 . asymmetric stretching mode (3) with $a^{\prime \prime}$.

Figure 1: symmetric stretching mode

Figure 2: symmetric bending mode

Figure 3: asymmetric stretching mode

### 8.1.2 PHHH

The number of vibrational degrees of freedom is $4 \cdot 3-6=6$. The point group of $\mathrm{PH}_{3}$ is $C_{3 v}$. Character table is given with

| $C_{3 v}$ | $E$ | $2 C_{3}(z)$ | $3 \sigma_{v}$ |
| :---: | :---: | :---: | :---: |
| $A_{1}$ | 1 | 1 | 1 |
| $A_{2}$ | 1 | 1 | -1 |
| $E$ | 2 | -1 | 0 |

With the reducible representation for cartesian coordinate transformation of

| $C_{3 v}$ | $E$ | $2 C_{3}(z)$ | $3 \sigma_{v}$ |
| :---: | :---: | :---: | :---: |
| $\Gamma$ | 12 | -4 | 4 |

This cannot be reduced, meaning I did something wrong.
Well, what we further would have liked to to is, reducing to vibrational modes

$$
\begin{aligned}
\Gamma^{v i b} & =\Gamma^{t o t}-\Gamma^{\text {trans }}-\Gamma^{\text {rot }} \\
& =\Gamma^{t o t}-\left(a_{1}+e\right)-\left(a_{2}+e\right)
\end{aligned}
$$

but since $\Gamma^{t o t}$ cannot be reduced using the irreducible representations (at least it would not contain $a_{1}+a_{2}+2 e$ ) we abort here and just do some minor work on the other tasks.

### 8.1.3 СНННН

There is a total number of $5 \cdot 3-6=9$ vibrational degrees of freedom. The point group of $\mathrm{CH}_{4}$ is $T_{d}$.

### 8.1.4 $\mathrm{CHCHCH}+$

We have $6 \cdot 3-6=12$ vibrational degrees of freedom.

