FREIE UNIVERSITÄT BERLIN

Fachbereich Physik

Übungen zur Vorlesung

''Einführung in die Physik der Atome und Moleküle I'' (SoSe 2007)

- Prof. Karsten Heyne -

Aufgabenblatt 6 vom 23.05.2007

Abgabe bei Dr. Henk Fidder, henk.fidder@physik.fu-berlin.de
vor Freitag 15.06.2007, 12.00 h.

Aufgabe 8—1 (2 Punkte)

Calculate for the hydrogen $2p_z$ orbital:

(a) the permanent dipole moment $< e\overrightarrow{r} >$

(b) the average distance of the electron from the nucleus $< r >$

Aufgabe 8—2 (2 Punkte)

(a) What are the term symbols for the $1s$ and $2p$ states of a hydrogen-like atom, i.e. one electron and a nucleus with charge +Ze. What are the energies and degeneracy of these states.

(b) Calculate the spontaneous emission rates for all electric dipole allowed transitions between $2p$ and $1s$ transition in these hydrogen-like atoms.

Aufgabe 8—3 (4 Punkte)

We consider an atom with two energy levels $|1\rangle$ and $|2\rangle$, with energies ϵ_1 and ϵ_2, coupled to a monochromatic radiation field by the interaction $H_{12} = Ve^{i\chi}$, with $\chi = k \cdot r - \omega t$ (V is real).
The Hamiltonian for this system can be represented as

$$H = \begin{pmatrix} \epsilon_1 & Ve^{i\chi} \\ Ve^{-i\chi} & \epsilon_2 \end{pmatrix}$$

(a) Despite the implicit time-dependence of χ, this Hamiltonian can be diagonalized. Calculate the eigenvalues ϵ_+ and ϵ_- of the two eigenstates of the diagonal matrix.

(b) Confirm that $|\Psi_+\rangle = cos\theta e^{-i\chi/2}|1\rangle + sin\theta e^{i\chi/2}|2\rangle$ and $|\Psi_-\rangle = -sin\theta e^{-i\chi/2}|1\rangle + cos\theta e^{i\chi/2}|2\rangle$ with θ defined by $tan 2\theta = \frac{2V}{\epsilon_1 - \epsilon_2}$ are the eigenstates of this Hamiltonian.

(c) Although we have two new eigenstates, we can continue to regard the quantum system as a separate two-level atom and the radiation field. The time-evolution operator can be constructed that allows to calculate the time-dependence of any linear combination of level $|1\rangle$ and $|2\rangle$ under the influence of the interaction.

This time-evolution operator is defined as

$U(t,t_0) = \left(|\Psi_+\rangle\langle\Psi_+|e^{\frac{-i}{\hbar}\epsilon_+(t-t_0)}\right) + \left(|\Psi_-\rangle\langle\Psi_-|e^{\frac{-i}{\hbar}\epsilon_-(t-t_0)}\right)$

If at $t_0 = 0$ the atom was in state $|1\rangle$, the time dependence of the population of level 2 is given by:

$P_{21}(t) = |\langle 2|U(t,t_0)|1\rangle|^2$

Derive the expression of this time-dependence (the Rabi formula).

Plot this time-dependence as a function of Ωt, where $\Omega = \frac{\left(4V^2+(\epsilon_1-\epsilon_2)^2\right)^{1/2}}{\hbar}$ is the Rabi frequency.

Aufgabe 8—4 (2 Punkte)

Sketch for sodium the energy level scheme for the electron configurations $[Ne]3s^1$ and $[Ne]3p^1$

(a) in absence of a magnetic field

(b) for the normal Zeeman effect (no L-S coupling)

(c) for the anomalous Zeeman effect (with L-S coupling)

Label the levels with the appropriate magnetic quantum numbers (m_ℓ, m_J).
Indicate all allowed transitions in the energy levels schemes.
Sketch the absorption spectrum as a function of energy for situations (a),(b)and(c).