Atom- und Molekülphysik SoSe 2007 (Prof. Heyne)

Übung Nr. 2:

Hand in latest: Friday, May 4, 12.00 h
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1) The time-averaged time-correlation function is

$$\overline{A(t)A(0)} \propto \int_0^\infty A(\tau)A(t+\tau)d\tau$$

Sketch and give a mathematical expression for the time-dependence of this correlation function for the following functions A(τ) :

a) A(τ) = e-γτ (1.5)

b) A(τ) = 1 for 0≤ τ ≤ 1; A(τ) = 0 for τ >1 (1.5)

2) The absorption spectrum can often be regarded as the convolution of the homogeneous line profile (Lorentzian: $f(\omega) = \frac{\Gamma_{hom}}{\Gamma_{hom}^2 + (\omega - \omega_0)^2}$), with an inhomogeneous broadening function (typically a Gaussian: $g(\omega) = e^{-\ln 2\left(\frac{2\omega}{\Gamma_{inh}}\right)^2}$).

An important mathematical property of convolutions is that its Fourier (or more general: Laplace) transform is equivalent to the product of the Fourier transforms of the two convoluted functions separately, i.e.

$$F(t)G(t) = \int_0^\infty e^{-i\omega t} f(\omega - \omega_0) g(\omega) d\omega$$

with: $F(t) = \int_0^\infty e^{-i\omega t} f(\omega) d\omega$, and $G(t) = \int_0^\infty e^{-i\omega t} g(\omega) d\omega$

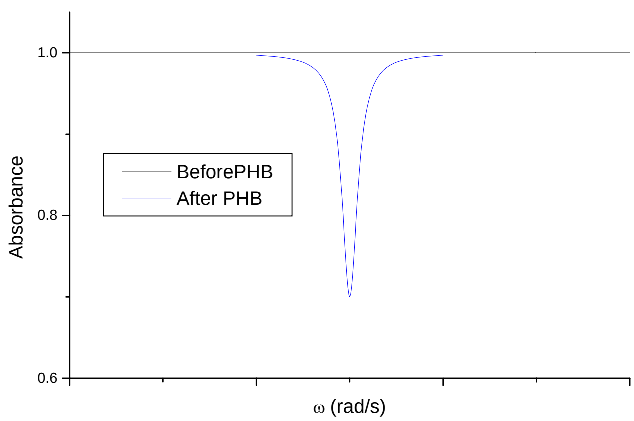
a) Calculate the function F(t)G(t) for the above described convolution of a homogeneous and an inhomogeneous line profile. (1.5)

b) The function F(t)G(t) shows how the macroscopic polarization of a sample, induced by excitation with an infinitely short laser pulse, decays in time. Sketch the decay function for the two extreme situations: I) $\Gamma_{hom} >> \Gamma_{inh}$
II) $\Gamma_{hom} << \Gamma_{inh}$ (1.5)

(1)

3) a) Mathematically the time-averaged time-correlation function is an example of convolution. Calculate the full width at halve maximum (FWHM) of the convolution of the normalized Lorentzian line shape function with itself. How does this FWHM relate to that of the normalized Lorentzian? (1.5)

b) A simple experiment for obtaining the homogeneous linewidth is photochemical hole-burning. We assume that the inhomogeneous line width is much broader than the homogeneous line width, so that on the frequency range of the homogeneous line width the absorption can be considered constant. Now the sample is irradiated with a nearly monochromatic laser (much narrower than the homogeneous line width), and absorption of this radiation leads to destruction of the absorbing molecules and the appearance of a Lorentzian shaped "hole" in the spectrum (see figure). The FWHM of this hole is 2 times the FWHM of the homogeneous line shape. Explain why the width is twice the homogeneous FWHM (instead of once). (1.5)



4) Using: $\delta A(t) = A(t) - < A(t) >_c$,

show that: $< \delta A(t) \delta(A(0) >_c =< A(t) A(0) >_c - < A >_c^2$ (eq.1.176 in the script)

(Hint: $< A(t) >_c =< A(0) >_c =< A >_c$) (1)